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THE PREDICTION OF HEAT TRANSFER IN THE WAKE OF CYLINDERS IN CROSSFLOW

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Abstract—A theory is presented which predicts heat transfer in the wake region of a cylinder in crossflow. The equations of Hiemenz describing laminar flow in the region of a stagnation point are modified to include an "apparent viscosity" term. The "apparent viscosity" is then related to the turbulent energy at the edge of the boundary layer in the wake by considering the laws of diffusion, dissipation and generation of turbulence. The solution of the equations, and the evaluation of the empirical constants introduced into the equations, is obtained by means of an analogue computer.

The theory is compared with experimental data and is in good agreement for a freestream Reynolds number and turbulence intensity range $5 \times 10^3 < Re_\infty < 3{\cdot}5 \times 10^4$ and 0·7 per cent $< Tu_\infty <$ 10 per cent.

NOMENCLATURE

Re_∞, freestream Reynolds number;
Nu, Nusselt number;
Tu, turbulence intensity;
x^*, y^*, coordinate parallel and normal to wall of cylinder;
x, y, $[\sqrt{(a/\nu)}]x^*, [\sqrt{(a/\nu)}]y^*$;
$\bar{u}^*, \bar{v}^*$, mean velocity parallel and normal to wall of cylinder;
$\bar{u}, \bar{v}$, $\bar{u}^*/\sqrt{(a\nu)}, \bar{v}^*/\sqrt{(a\nu)}$;
ρ, density;
p^*, static pressure;
p, $p^*/a\nu$;
p_s^*, total head pressure;
ν, kinematic viscosity;
ν_t, turbulent kinematic viscosity;
T^*, temperature;
T, $\dfrac{T_w^* - T^*}{T_w^* - T_G^*}$;
a, $\dfrac{4\bar{u}_G}{D}$;
D, cylinder diameter;
$\overline{U}, \overline{V}$, mean velocity parallel and normal to wall at edge of boundary layer;
$\phi(y), F(y)$, functions in equations (9), (10) and (11);
σ, Prandtl number;
σ_t, turbulent Prandtl number;
k, turbulent energy, $\frac{1}{2}\{\overline{u'^2}+\overline{v'^2}+\overline{w'^2}\}$;
$\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$, mean squared fluctuating velocity components;
τ, shear stress in equation (16);
a, b, constants in equation (17);
l_μ, l_D, functions in equation (18);
$c_\mu, C_D, \sigma_{k,t}$, constants in equation (18);
R, $\dfrac{k^{1/2}y^*}{\nu}$;
R_{osc}, $\dfrac{k_G^{1/2}y_G^*}{\nu}$;
η, y^*/y_G^*;
n, $1-\dfrac{1}{\varepsilon}\eta$;
U, $\bar{u}^*/\bar{u}_G^*$;
K, k/k_G;
A_1, A_2, A_3, B, constants in equation (21);
N, function in equation (21);
ε, constant in equation (22).

Superscript
$*$, having dimensions.

Subscript
G, condition at edge of boundary layer;
w, condition at wall.

1. INTRODUCTION

THERE is no theory currently available which can adequately predict local heat transfer in the wake region of bluff bodies and in particular in the wakes of cylinders in crossflow. Although there is a large amount of heat-transfer data available for such regions a large variation has been shown to exist. Richardson [1] has devised an empirical relationship covering a range of data and this appears to follow an $Re_\infty^{2/3}$ relationship. Some of the discrepancies may be accounted for by the effect of freestream turbulence intensity and scale as demonstrated by Zijnen [2] and which have also been confirmed by Petrie and Simpson [3]. A comparison of any theory with experimental data is therefore difficult. The main difficulty lies in the basic lack of understanding of the fluid mechanics of this region since the flow is characteristically unstable and the problem is likely to be a three dimensional one although simplification to a two dimensional case is often assumed. It is on this basis that mathematical models to include the intensity and scale of turbulence are usually derived.

One attempt has been made to predict heat transfer in such a region by Leontev and Riagen [4] by extending the analytical methods of boundary-layer theory. It is assumed that the surface of the cylinder is wetted by a relatively stable turbulent return flow and that there is a growth of a layer starting at the rear stagnation point which obeys boundary-layer theory. By considering the steady, incompressible conserved property equation and empirical relationships between Stanton number and thermal boundary-layer thickness, equations for local Nusselt number around the wall have been derived for the case of constant surface heat flux and constant temperature surface. Both are dependent upon a freestream Reynolds number factor raised to the half power which is in contradiction to some of the experimental evidence. Since the region of interest is characteristically one of large fluctuations rather than large velocities the exclusion of this fact in any theory suggests it to be inadequate.

To relate the fluctuating fluid motion to the heat transfer in separated flows it is necessary to investigate the decay of turbulence in the region of the wall. Spalding [5] has considered this problem on a one dimensional basis and has derived a power law relationship between the Stanton number and Reynolds number expressing the law of heat transfer for a wall adjacent to a region of separated flow. The derivation is based on Prandtl's proposals for the laws of dissipation, diffusion and generation of turbulent kinetic energy. One of the characteristic features of separated flows is that the locations of maximum shear stress are remote from the wall. The turbulence which is generated in the remote high shear region of a separated flow must be conveyed to the vicinity of the wall by the action of convection and diffusion; the turbulence intensity near the wall which is a main determinant of heat transfer, is governed by the interaction of these two factors together with turbulence dissipation. By relying heavily on experimental data for flow in a separated region a law is derived in the form which shows a dependence of Nusselt number to the Reynolds number to the 0·6 power.

A recent publication by Petrie and Simpson [3] provides the results of an experimental study of the effect of mainstream turbulence on heat transfer from a cylinder in crossflow, with particular attention being paid to the wake region of the cylinder. The data are obtained in the ranges $5 \times 10^3 < Re_\infty < 3{\cdot}5 \times 10^4$ and $0{\cdot}7$ per cent $< Tu < 10$ per cent. Using an electrically heated constant heat flux cylinder and measuring local heat transfer, improvements up to 100 per cent in the wake region were realised. A single wire probe was used to study the flow in this region, mean and fluctuating components of velocity normal to and around the wall being measured. Weak gradients of pressure, temperature and velocities around the wall were found to exist for $\pm 40°$ from the rear stagnation point and the boundary-layer thickness was also apparently constant in this region. By using the RMS fluctuating velocity component as a measure of the turbulent kinetic energy a correlation between Nusselt number and turbulent energy was obtained. All the experimental data are presented in Figs. 2, 3 and 4.

By using the relationships between shear stress and turbulent energy similar to those discussed by Spalding [5], a mathematical relationship is derived to predict heat transfer in the wake region which compares favourably with the experimental measurements. An analogue computer facility has been used to solve the equation set and the empirical constants occurring in the equations are evaluated using the fluid mechanics experimental data.

2. THE PREDICTION OF HEAT TRANSFER

2.1. *Formulation of equations*

From the available experimental data, several simplifying assumptions may be made to assist in the prediction of heat transfer from the fluid mechanics. Because of the weak gradients around the wall in the region of the rear stagnation point, it was considered sufficient at this stage to derive a relationship for conditions at the rear stagnation point. The steady flow solution at a stagnation point has already been solved by Hiemenz for laminar flow [8]. By modifying his equations to include the effect the turbulence in the form of an apparent viscosity term and by considering this term as a function of the turbulent kinetic energy it is then possible to relate the variation of viscosity to the diffusion, dissipation and generation of turbulence near the wall.

As is common practice the fluctuating stress terms in the Navier–Stokes equations can be lumped together into an "apparent" or eddy viscosity term and the equations can then be written in the form

$$\bar{u}^* \frac{\partial \bar{u}^*}{\partial x^*} + \bar{v}^* \frac{\partial \bar{u}^*}{\partial y^*}$$
$$= -\frac{1}{\rho}\frac{\partial p^*}{\partial x^*} + \frac{\partial}{\partial x^*}\left[(\nu+\nu_t)\frac{\partial \bar{u}^*}{\partial x^*}\right] + \frac{\partial}{\partial y^*}\left[(\nu+\nu_t)\frac{\partial \bar{u}^*}{\partial y^*}\right] \quad (1)$$

$$\bar{u}^* \frac{\partial \bar{v}^*}{\partial x^*} + \bar{v}^* \frac{\partial \bar{v}^*}{\partial y^*} = -\frac{1}{\rho}\frac{\partial p^*}{\partial y^*} + \frac{\partial}{\partial x^*}\left[(v+v_t)\frac{\partial \bar{v}^*}{\partial x^*}\right] + \frac{\partial}{\partial y^*}\left[(v+v_t)\frac{\partial \bar{v}^*}{\partial y^*}\right] \quad (2)$$

$$\bar{u}^* \frac{\partial T^*}{\partial x^*} + \bar{v}^* \frac{\partial T^*}{\partial y^*} = \frac{\partial}{\partial x^*}\left[\left(\frac{v}{\sigma}+\frac{v_t}{\sigma_T}\right)\frac{\partial T^*}{\partial x^*}\right] + \frac{\partial}{\partial y^*}\left[\left(\frac{v}{\sigma}+\frac{v_t}{\sigma_T}\right)\frac{\partial T^*}{\partial y^*}\right] \quad (3)$$

where the * indicates having dimensions.

The equations can then be written in a non-dimensional form using the same groups as Hiemenz, that is,

$$x = [\sqrt{(a/v)}]x^*, \quad \bar{u}^* = [\sqrt{(av)}]\bar{u}, \quad T = \frac{T_w^* - T^*}{T_w^* - T_G^*}$$
$$y = [\sqrt{(a/v)}]y^*, \quad \bar{v}^* = [\sqrt{(av)}]\bar{v}, \quad \frac{p^*}{\rho} = av\frac{p}{\rho} \quad (4)$$

where $a = 4\bar{U}_G/D$ = a reciprocal time constant.

In frictionless potential flow the velocity distribution in the neighbourhood of the stagnation point is given by

$$\bar{U} = ax, \qquad \bar{V} = -ay \quad (5)$$

and if p_s denotes the stagnation pressure and p the pressure at any arbitrary point then

$$p_s^* - p^* = \tfrac{1}{2}\rho a^2(x^2+y^2). \quad (6)$$

For laminar flow it is assumed that

$$\bar{u} = x\phi'(y); \qquad \bar{v} = -\phi(y) \quad (7)$$

where the prime denotes differentiation with respect to y. Thus

$$p_s^* - p^* = \tfrac{1}{2}a^2\rho(x^2 + F(y)) \quad (8)$$

where $\phi(y)$ and $F(y)$ are both functions of y.

By non-dimensionalising equations (1), (2), (3) and substituting expressions (7) and (8) into these equations we have

$$\frac{d}{dy}\left[\left(1+\frac{v_t}{v}\right)\phi''\right] + \phi\phi'' - \phi'^2 + 1 = 0 \quad (9)$$

$$\frac{d}{dy}\left[\left(1+\frac{v_t}{v}\right)\phi''\right] - \phi\phi'' + \tfrac{1}{2}F' = 0 \quad (10)$$

$$\frac{d}{dy}\left[\left(1+\frac{\sigma}{\sigma_t}\cdot\frac{v_t}{v}\right)T'\right] + \sigma\phi T' = 0. \quad (11)$$

The corresponding boundary conditions are then

$$y = 0: \quad \phi = 0, \quad \phi' = 0, \quad F = 0, \quad T = 1$$
$$y = \infty: \quad \phi' = 1, \quad \phi'' = 0, \quad T = 0.$$

Since equations (9) and (10) are uncoupled, if the pressure distribution is not required, equations (9) and (11) may be solved simultaneously to obtain temperature profiles. For the case of laminar flow these equations reduce to the familiar form derived by Hiemenz.

$$\phi''' + \phi\phi'' - \phi'^2 + 1 = 0 \quad (12)$$

$$\phi''' - \phi\phi'' + \tfrac{1}{2}F' = 0 \quad (13)$$

$$T'' + \sigma\phi T' = 0. \quad (14)$$

2.2. *Turbulent energy equation*

Most present day methods involving calculations with turbulence assume that the shear stress at a point is dependent upon the mean velocity gradient at that point. It has been shown by Bradshaw [7] however, that there is a much closer connection between the shear stress and the parameters which describe the turbulence structure, the latter being governed by the turbulent kinetic energy equation. One form of this equation is based on Prandtl's proposals for the laws of dissipation, generation and diffusion of turbulence.

By defining the turbulent kinetic energy k as

$$k = \tfrac{1}{2}\{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\} \quad (15)$$

these relationships are then written as

Rate of dissipation per unit volume

$$= a\rho\frac{k^{3/2}}{y^*}$$

Rate of generation per unit volume

$$= \tau\frac{d\bar{u}^*}{dy^*}$$

Rate of diffusion into unit volume

$$= b\rho\frac{d}{dy^*}\left(k^{1/2}y^*\frac{dk}{dy^*}\right) \quad (16)$$

where

$$\frac{\tau}{\rho} = v_t\frac{d\bar{u}^*}{dy^*}, \qquad v_t = ck^{1/2}y^*.$$

Thus in a turbulent region

$$a\frac{k^{3/2}}{y^*} - b\frac{d}{dy^*}\left(k^{1/2}y^*\frac{dk}{dy^*}\right) - \frac{\tau}{\rho}\cdot\frac{d\bar{u}^*}{dy^*} = 0. \quad (17)$$

Based upon the available experimental evidence [3] that the turbulent energy gradients are weak around the wall, convection terms have been neglected. It is now required to relate the decay of turbulence in this region to that in the viscous region close to the wall. This is hampered by lack of knowledge and experimental data in such a region and one has to rely on many empirical relationships. In order to have one equation which describes the decay of turbulence in both regions, Wolfshtein [6] has modified equation (17) to include two exponential terms l_μ and l_D which are claimed to be valid in the viscous layer in the wall.

Although this may not be of high accuracy it does simplify the solution procedure and is likely to be adequate for the present task. Equation (17) then becomes

$$\left.\begin{aligned} &\frac{d}{dy^*}\left[\left(\nu+\frac{\nu_t}{\sigma_{k,t}}\right)\frac{dk}{dy^*}\right]+\nu_t\left(\frac{d\bar{u}}{dy^*}\right)^2-\frac{c_D k^{3/2}}{l_D}=0\\ &\text{where}\quad \nu_t = c_\mu k^{1/2} l_\mu\\ &\qquad l_\mu = y^*(1-e^{-A_\mu R})\\ &\qquad l_D = y^*(1-e^{-A_D R});\quad R=\frac{k^{1/2}y^*}{\nu}. \end{aligned}\right\} \tag{18}$$

The most convenient way of non-dimensionalising this equation is to consider the boundary-layer thickness and the velocity and turbulence levels at the edge of the boundary layer thus,

$$\eta=\frac{y^*}{y_G^*};\qquad U=\frac{\bar{u}^*}{\bar{u}_G^*};\qquad K=\frac{k}{k_G}. \tag{19}$$

Equation (18) is then written as

$$\left.\begin{aligned} &\frac{d}{d\eta}\left[\left(1+\frac{1}{\sigma_{k,t}}\cdot\frac{\nu_t}{\nu}\right)\frac{dK}{d\eta}\right]+\frac{\bar{u}^{*2}}{k_G}\cdot\frac{\nu_t}{\nu}\left(\frac{dU}{d\eta}\right)^2\\ &\qquad -\frac{k_G^{1/2}y_G^*}{\nu}\cdot\frac{C_D K^{3/2}}{\eta(1-e^{-A_D R})}=0\\ &\frac{\nu_t}{\nu}=\frac{k_G^{1/2}y_G^*}{\nu}\cdot c_\mu K^{1/2}\eta(1-e^{-A_\mu R});\\ &\qquad R=\frac{k_G^{1/2}y_G^*}{\nu}K^{1/2}\eta. \end{aligned}\right\} \tag{20}$$

The equations which now require simultaneous solutions can be simplified to

$$\left.\begin{aligned} &\frac{d}{d\eta}\left[\left(\frac{1}{R_{osc}}+A_1N\right)\frac{dK}{d\eta}\right]\\ &\qquad +BN[\phi'']^2-\frac{C_D K^{3/2}}{\eta(1-e^{-A_D R})}=0\\ &\frac{d}{dy}[(1+A_3N)\phi'']+\phi\phi''-\phi'^2+1=0\\ &\frac{d}{dy}[(1+A_2N)T']+\sigma\phi T'=0\\ &\text{where}\\ &N=K^{1/2}\eta(1-e^{-A_\mu R});\qquad R_{osc}=\frac{k_G^{1/2}y_G^*}{\nu}\\ &A_1=\frac{c_\mu}{\sigma_{k,t}};\qquad A_2=\frac{\sigma c_\mu R_{osc}}{\sigma_T};\\ &A_3=c_\mu R_{osc};\qquad B=\frac{c_\mu \bar{u}_G^2}{k_G}. \end{aligned}\right\} \tag{21}$$

The facilities available indicated that the most efficient way to obtain a solution was by means of an analogue computer.

Before attempting a solution two slight modifications to the equations are necessary. Firstly, the non-dimensional procedure used by Hiemenz does not provide a suitable measure of the boundary-layer thickness and therefore y is modified to be compatible with η thus

$$\eta=\frac{y^*}{y_G^*}=\varepsilon y=\varepsilon\frac{y^*}{\sqrt{(\nu/a)}}. \tag{22}$$

Secondly, because of a computing technique, it is necessary to start the simultaneous solution of equations (21) at $\eta = 1$, that is, at the edge of the boundary layer and therefore a new independent variable is necessary which is a function of y

$$n = 1-\frac{1}{\varepsilon}\eta = 1-y.$$

Thus

$$\frac{d}{d\eta}=-\frac{1}{\varepsilon}\cdot\frac{d}{dn}$$

and

$$\frac{d^2}{d\eta^2}=\frac{1}{\varepsilon^2}\cdot\frac{d^2}{dn^2}.$$

The final equations are then

$$\left.\begin{aligned} &\phi_n''=-\frac{\varepsilon}{A_3N}\int\{\phi_n'^2-\phi_n\phi_n''-\varepsilon^2\}\,dn &&\text{(a)}\\ &T_n'=\frac{\varepsilon}{A_2N}\int\sigma T_n'\phi\,dn &&\text{(b)}\\ &\frac{dK}{dn}=\frac{1}{A_1N}\int\left\{\frac{C_D K^{3/2}}{(1-n)(1-e^{-DK^{1/2}(1-n)})}\right.\\ &\qquad\qquad\left.-B[\phi_n'']^2\right\}dn &&\text{(c)}\\ &N=K^{1/2}(1-n)(1-e^{-EK^{1/2}(1-n)})\\ &E=A_\mu R_{osc};\qquad D=A_D R_{osc} \end{aligned}\right\} \tag{23}$$

with the following boundary conditions

$$\eta=0:\quad K=0;\ \phi_n'=0;\ \phi_n=0;\ T_n=1$$
$$\eta=1:\quad K=1;\ \phi_n'\approx 1;\ T_n\approx 0.$$

An iterative procedure is necessary to solve equations (23a) and (23b). This is effected by adjusting the initial gradient of each equation to satisfy all boundary conditions. Initial tests proved that the turbulence generation term had little effect on the solution of the turbulent energy equation thus simplifying the solution procedure. An investigation of the effect of the exponential terms l_μ, l_D indicated that for the range of other constants involved the numerical values of E and D made little difference to the solution and therefore both constants were fixed at 0·5.

The experimental data has indicated that the temperature gradient around the wall $dT/d\theta$ at the rear stagnation point is weak and therefore the assumption of constant temperature is valid. Thus

$$q_w = \alpha \frac{\Delta T}{y_G^*}\left(\frac{dT}{d\eta}\right)_{\eta=0}$$

where

$$\Delta T = T_w - T_G.$$

Thus

$$Nu = \frac{D}{y_G^*}\left(\frac{dT}{d\eta}\right)_{\eta=0.}$$

It was then a simple matter after allowing for these simplifications to evaluate the temperature gradient at the wall and hence the local heat transfer.

3. RESULTS AND DISCUSSION

It was necessary to compare the accuracy of the solution procedure by comparing the results with known solutions. This was done satisfactorily for the special case of $\nu_t = 0$, that is for laminar flow, with the numerical solution of Hiemenz. A comparison of the velocity gradient at the wall, was as follows:

$$\text{Present solution} \quad \phi''_{\eta=0} = 1{\cdot}227$$
$$\text{Hiemenz solution} \quad \phi''_{\eta=0} = 1{\cdot}233.$$

The difference of 0·5 per cent was considered extremely good and certainly sufficiently accurate for the present analysis. The laminar flow solutions are shown in Fig. 1.

Several constants occur in the equation set which can be adjusted to fit the theory to the experimental data. Since it is desirable to choose a group of constants which are generally applicable to a number of fluid flow problems, it was decided to use initially the same group as Wolfshtein [6] which were based on a turbulent boundary layer on a flat plate. These were as follows:

$$C_\mu = 0{\cdot}22 \qquad C_D = 0{\cdot}416 \qquad \sigma_{k,t} = 1{\cdot}53$$
$$A_\mu = 0{\cdot}016 \qquad A_D = 0{\cdot}263 \qquad \sigma_T = 0{\cdot}9.$$

A laminar Prandtl number of 0·7 was used throughout and an initial value of $\varepsilon = 2{\cdot}0$ was chosen based on the laminar value at a stagnation point (Hiemenz solution).

Not surprisingly this group was unsuitable and several alterations were required. The solutions were not strongly dependent upon l_μ and l_D and therefore the constants A_μ and A_D were retained at the above values. Agreement between theory and experiment was obtained by reducing σ_t but this was considered unreasonable since values less than the laminar value of 0·7 were required, therefore σ_t was retained at 0·9. The turbulent energy within the boundary layer is dependent largely upon the diffusion into the layer. It seems reasonable to expect this to be a function of the maximum turbulent energy, in this case located at the edge of the boundary layer. Alteration of the constant c_μ in the diffusion term of equation (21a) to become a variable, $c_\mu = f(R_{osc})$, resulted in a theoretical relationship between Nusselt number and turbulent energy which is in reasonably good agreement with the experimental data, a comparison being made in Fig. 2.

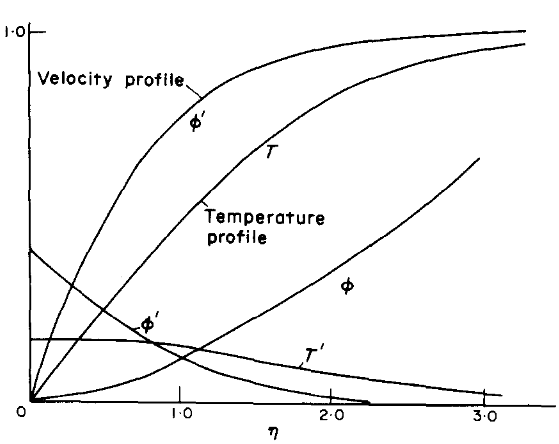

FIG. 1. Solutions of equations for laminar boundary-layer case (Hiemenz solution).

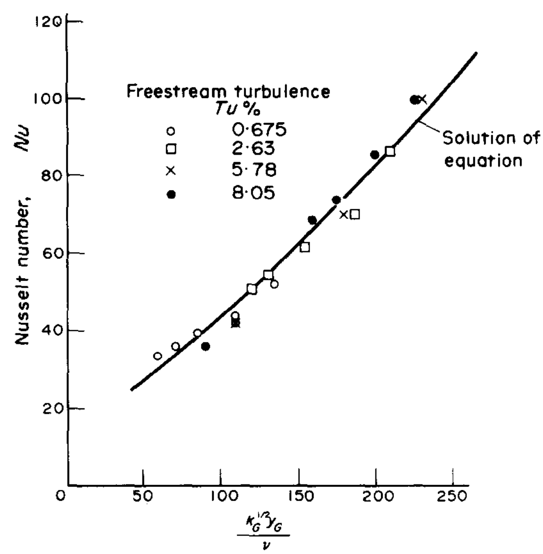

FIG. 2. Prediction of heat transfer at rear stagnation point: a comparison with experimental data.

A simple power law relationship for this new variable was found to be adequate. The best fit being produced by

$$c_\mu = 0{\cdot}051\left(\frac{k_G^{1/2} y_G^*}{\nu}\right)^{0\cdot5}$$

The alteration of the diffusion term to include the effect of the maximum turbulent energy is in agreement with the work of Bradshaw [7].

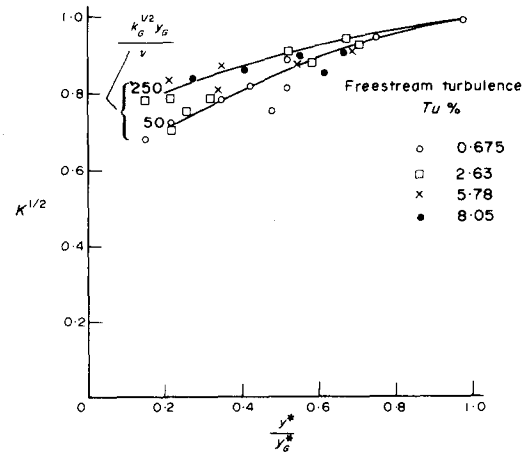

FIG. 3. Non-dimensional fluctuating velocity profile at rear stagnation point: a comparison with experimental data.

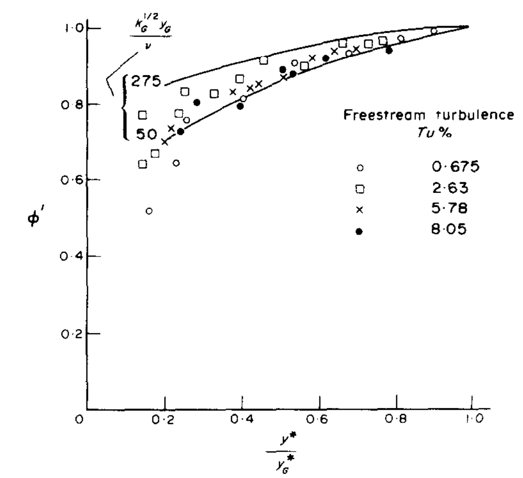

FIG. 4. A comparison of theoretical and experimental mean velocity profiles for $\varepsilon = 3{\cdot}0$.

Once the value of c_μ is established the remaining constants can be adjusted accordingly. Examination of the equation (21a) shows that there is no need to alter $\sigma_{k,t}$ and C_D. C_D was therefore chosen and reduced to 0·135 which gave the best agreement for the RMS fluctuating velocity profiles. Because of the amount of scatter of experimental data due to freestream turbulence effects, the range of $k_G^{1/2} y_G^*/\nu$ to include this scatter is indicated in Fig. 3. This range, $0 < k_G^{1/2} y_G^*/\nu < 250$ is within that measured by experiment. Finally, by increasing $\varepsilon = 3{\cdot}0$, implying a reduction in the thickness of the boundary layer a satisfactory agreement is indicated in Fig. 4 was obtained between the predicted and measured mean velocity profiles.

While there is still a heavy reliance upon experimental data, which will remain necessary until there is a better understanding of the fluid mechanics of this region, the theory provides a general framework on which to examine the fluid mechanic parameters most likely to affect the heat transfer.

CONCLUSIONS

1. The mathematical model chosen to predict the heat transfer in the wake region of a cylinder is satisfactory so far as can be determined in the light of available experimental data.
2. On the basis of the available information the most suitable constants to be used in the equation set are:

$$c_\mu = 0{\cdot}051\left(\frac{k_G^{1/2} y_G^*}{\nu}\right)^{0\cdot5} \quad C_D = 0{\cdot}135 \quad \sigma_{k,t} = 1{\cdot}53$$

$$\varepsilon = 3{\cdot}0 \quad A_\mu = 0{\cdot}016; \quad A_D = 0{\cdot}263; \quad \sigma_t = 0{\cdot}9.$$

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PREVISION DU TRANSFERT THERMIQUE DANS LE SILLAGE DE CYLINDRE EN ATTAQUE FRONTALE

Résumé—On présente une théorie qui donne le transfert thermique dans la région de sillage d'un cylindre en attaque frontale. Les équations de Hiemen qui décrivent l'écoulement laminaire dans la région du point d'arrêt, sont modifiées pour inclure un terme de "viscosité apparente". La "viscosité apparente" est reliée à l'énergie turbulente à la frontière de la couche limite dans le sillage en considérant les lois de la diffusion, de la dissipation et de la génération de la turbulence. La solution des équations et l'évaluation des constantes empiriques introduites dans les équations sont obtenues au moyen d'un calculateur analytique.

La théorie est comparée avec les résultats expérimentaux et l'accord est bon pour un nombre de Reynolds tel que $5 \times 10^3 < Re_\infty < 3,5 \times 10^4$ et une intensité de turbulence 0,7 pour cent $< Tu_\infty <$ 10 pour cent.

DIE VORAUSBESTIMMUNG DES WÄRMEÜBERGANGS IN DEN ABLÖSEWIRBELN QUERANGESTRÖMTER ZYLINDER

Zusammenfassung—Es wird eine Theorie angegeben, mit der der Wärmeübergang im Totwassergebiet eines querangeströmten Zylinders vorausberechnet werden kann. Durch Einführen eines "Scheinviskositäts"—Terms wurden die Gleichungen von Hiemenz, die die laminare Strömung im Bereich des Satupunkts beschreiben, modifiziert. Die "Scheinviskosität" wird dann mit der Turbulenzenergie am Rand der Grenzschicht im Wirbelgebiet unter Berücksichtigung der Gesetzmäßigkeiten der Diffusion, der Dissipation und der Erzeugung von Turbulenz verknüpft. Die Lösung der Gleichungen und die Bestimmung der empirischen Konstanten in den Gleichungen werden mit einem Analogrechner durchgeführt. Die Theorie stimmt mit experimentellen Ergebnissen für Freistrom-Reynolds-Zahlen zwischen $5 \times 10^3 < Re_\infty < 3,5 \times 10^4$ und Turbulenzgrade zwischen $0,7\% < Tu_\infty < 10\%$ gut überein.

РАСЧЁТ ПЕРЕНОСА ТЕПЛА В СЛЕДЕ ЦИЛИНДРОВ ПРИ ПОПЕРЕЧНОМ ОБТЕКАНИИ

Аннотация — Предложена теория, позволяющая рассчитать перенос тепла в следе поперечно обтекаемого цилиндра. В уравнение Хименца, описывающее ламинарное течение в области критической точки, введена «кажущаяся вязкость». «Кажущаяся вязкость» затем связывается с турбулентной энергией на внешней границе пограничного слоя в следе и дополняется законами диффузии, диссипации и генерирования турбулентности. Решение этих уравнений и определение эмпирических констант в уравнениях выполнены с помощью аналоговой машины. Сравнение теоретических данных с экспериментальными результатами обнаруживает хорошее согласие для значений числа Рейнольдса свободного течения $5 \times 10^3 < Re_\infty < 3,5 \times 10^4$ в диапазоне изменения интенсивности турбулентности $0,7\% < Tu_\infty < 10\%$.